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Conditional cooperator enhances institutional punishment in public goods game

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ABSTRACT

The role of incentive institutions on promoting cooperation in public goods game (PGG) has attracted much attention. Theoretical studies based on Nash equilibrium analysis predict that the punishment effect is often stronger than the reward effect. Although this result is confirmed by empirical studies, subjects do not always play these rational strategies. Recent experiments indicate that most subjects in PGGs are conditional cooperators who tend to contribute the group average. In this paper, we consider PGGs with three types of subjects, namely, cooperators, defectors, and conditional cooperators. Evolutionary game method is applied to investigate how conditional cooperators affect the effectiveness of different types of incentives. Overall, having conditional cooperators cannot lead to a higher contribution level in the standard PGG or the PGG with institutional rewards. However, they can enhance the effectiveness of institutional punishment, where a high contribution level can be maintained even for small punishments. As a consequence, in PGGs with conditional cooperators, punishment always leads to a higher contribution rate than do rewards. Numerical analysis indicates that this result is robust to errors in decision making.

1. Introduction

Sustaining cooperation in groups of unrelated individuals is a challenging question [1–3]. In human society, a solution for this problem is to establish institutions that reward cooperators and punish exploiters. The role of incentives in boosting cooperation is a well-studied topic in economics and evolutionary biology (see [4–6] for three review papers). Most of previous studies are based on Nash equilibrium (NE) analysis and evolutionary game analysis, where individuals tend to choose the strategy with a higher payoff [7–19]. These studies revealed that effect of institutional incentives on collective actions relies on the size of the incentive. If the incentive size is small, then both reward and punishment cannot promote cooperation, and free-riding is the only evolutionarliy stable strategy (ESS). If the incentive is large, then cooperation becomes a unique ESS for the both types of incentives. Finally, if the incentive is intermediate, then the effects of reward and punishment are not equivalent. In the case of punishment, the cooperative state and defective state are bistable, which means that punishment can eliminate extremely selfish behaviors in a cooperative population but does not work for the selfish population. In contrast, reward can cause only the stable coexistence of cooperators and defectors. Overall, an intermediate size

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of punishment is sufficient to support full contribution, but reward can maintain stable cooperation only for a large incentive size. Empirical studies on public goods game (PGG) with incentives have confirmed that both institutional reward and institutional punishment can curb free riding and that the punishment effect is often stronger than the reward effect [20–28]. Although Nash equilibrium analysis correctly predicts which incentives are better at promoting cooperation, individuals do not always play payoff-maximizing strategies. Recent experiments observed that most subjects in PGGs with incentives are conditional cooperators (also called conformists), where they tend to decrease (or increase) their contributions if they contributed more (or less) than the group average [25,27,28].

It has been shown that conditional cooperators play a predominant role in social dilemma games such as the repeated Prisoner's Dilemma (PD) game and repeated PGG. In the repeated PD, tit-for-tat (TFT) should be considered one of the most famous conditional cooperative strategies. If most subjects in the population adopt this strategy, then full cooperation can be evolutionarily stable via direct reciprocity [2,3]. Moreover, recent studies showed that conditional cooperators can also enhance cooperation in network PD game [29,30]. In the repeated PGG, a conditional cooperator tends to choose the group average contribution [31–36]. Empirical studies indicated that approximately half of the individuals in PGGs can be classified as conditional cooperators [32,34–36]. The existence of conditional cooperators can help to maintain a high contribution level in smaller groups and slow the decrease in the average contribution in larger groups [28,33,34,36,37]. Recent experiments have revealed that conditional cooperators are also widespread in PGGs with institutional incentives [25,27,28]. Although statistical analysis and numerical simulations showed that the existence of conditional cooperators may help to enhance cooperation [27,28], a systematic study of the effect of conditional cooperators on PGGs with different types of incentives is still lacking.

In this paper, we consider PGGs with three types of subjects, namely, cooperators, defectors, and conditional cooperators. Naturally, a cooperator always contributes all of his/her initial endowment, and a defector always contributes nothing. In contrast, the behaviour of a conditional cooperator depends on the population composition, where the more cooperators and fewer defectors there are, the more the conditional cooperator contributes [32,34]. Suppose that there is a third-party institution that can reward and punish players based on their contributions, where full contributors will be rewarded and free riders will be punished [8–13,16,18,21,23,26,38]. To better simulate reality, we consider that individuals are boundedly rational and do not always choose the optimal strategy. Following previous studies, we assume that players update their strategies by imitating individuals with higher payoffs and may make mistakes in strategy choosing [16,39–41]. This strategy updating process can be formulated as the replicator-mutator dynamics [47]. To investigate the effect of conditional cooperators on different types of incentives, we compare the average contribution levels at the stable equilibria. Overall, conditional cooperators cannot lead to a higher cooperation level for standard PGGs and PGGs with institutional rewards. In contrast, they can enhance the effectiveness of institutional punishment, where a high cooperation level can be maintained even for small punishments.

2. Model and analysis

2.1. PGG with conditional cooperators

Consider an infinitely large population with three types of players, cooperators, defectors, and conditional cooperators, where their frequencies are x, y, and z, respectively. In each time step, n individuals are randomly chosen from the population to form a PGG [8,9,16]. In the PGG, each player can contribute an amount between 0 and c to a common pool. The total contributions to common pool will be multiplied by a factor r with $1 < r < n$ and split evenly among all n group members. Thus, a rational player intends to contribute nothing, but the social optimal is that all players contribute their total endowment. Intuitively, r measures the benefit-to-cost ratio of the cooperative behaviours, where a larger (or a smaller) r decreases (or increases) the social dilemma strength. We note that the PGG is closely related to a special case of the Prisoner's Dilemma game, the Donation game [3,42–46]. In the Donation game, cooperation means that confers a benefit of b at a cost of c, and the dilemma strength of the game can be measure by $\frac{c}{b-c}$ [42–44,46]. Anomalous to the PGG, a larger benefit-to-cost ratio decreases the strength of the dilemma.

In line with previous studies, we assume that a cooperator always contributes c, and a defector always contributes 0 [2,3,8,9,16]. In contrast, the behaviour of a conditional cooperator depends on the population composition. For simplicity, we assume that the contribution of a conditional cooperator equals the population average, which has the form $c\frac{x}{x+y}$. (see Section 3 for more discussions). Furthermore, if the population consists of conditional cooperators only, then they act as cooperators and always contribute c. We now calculate the expected payoffs for the three types of players. In a randomly formed PGG, the probability that a focal player has n_C cooperators, n_D defectors, and n_{CC} conditional cooperators among the $n-1$ co-players is

$$\binom{n-1}{n_C}\binom{n-1-n_C}{n_D}x^{n_C}y^{n_D}z^{n_{CC}}, \tag{1}$$

where $n_C + n_D + n_{CC} = n - 1$. If the focal player is a cooperator, then his/her payoff is

$$\frac{cr}{n}\left(n_C + 1 + \frac{x}{x+y}n_{CC}\right) - c. \tag{2}$$

If the focal player is a defector, then the payoff is

$$\frac{cr}{n}\Big(n_C + \frac{x}{x+y}n_{CC}\Big). \tag{3}$$

Finally, if he/she is a conditional cooperator, then the payoff is

$$\frac{cr}{n}\Big(n_C + \frac{x}{x+y}(n_{CC}+1)\Big) - c\frac{x}{x+y}. \tag{4}$$

We denote the expected payoffs of a cooperator, a defector, and a conditional cooperator by P_C, P_D, and P_{CC}, respectively. Thus, in a randomly formed PGG with $n-1$ co-players,

$$\begin{aligned}
P_C &= \sum_{n_C=0}^{n-1}\sum_{n_D=0}^{n-1-n_C}\binom{n-1}{n_C}\binom{n-1-n_C}{n_D}x^{n_C}y^{n_D}z^{n_{CC}}\Big(\tfrac{cr}{n}\big(n_C+1+\tfrac{x}{x+y}n_{CC}\big)-c\Big)\\
&= \tfrac{rc(n-1)}{n}\tfrac{x}{x+y} - c + \tfrac{cr}{n},\\
P_D &= \sum_{n_C=0}^{n-1}\sum_{n_D=0}^{n-1-n_C}\binom{n-1}{n_C}\binom{n-1-n_C}{n_D}x^{n_C}y^{n_D}z^{n_{CC}}\tfrac{cr}{n}\big(n_C+\tfrac{x}{x+y}n_{CC}\big)\\
&= \tfrac{rc(n-1)}{n}\tfrac{x}{x+y},\\
P_{CC} &= \sum_{n_C=0}^{n-1}\sum_{n_D=0}^{n-1-n_C}\binom{n-1}{n_C}\binom{n-1-n_C}{n_D}x^{n_C}y^{n_D}z^{n_{CC}}\Big(\tfrac{cr}{n}\big(n_C+\tfrac{x}{x+y}(n_{CC}+1)\big)-c\tfrac{x}{x+y}\Big)\\
&= c(r-1)\tfrac{x}{x+y}.
\end{aligned} \tag{5}$$

Furthermore, the population average payoff is $\bar{P} = xP_C + yP_D + zP_{CC} = c(r-1)\frac{x}{x+y}$. Unsurprisingly, in the standard PGG, the expected payoff for a condition cooperator is the same as the group average. We further define the fitness of the three types of players by f_C, f_D, and f_{CC}, respectively, where the fitness has the form $f_X = \omega P_X + 1 - \omega$ with $X = \{C, D, CC\}$. The parameter $\omega \in [0, 1]$ measures the intensity of selection, where the fitness is not affected by the payoff when $\omega = 0$ and is entirely up to the payoff when $\omega = 1$ [2,47].

Following previous studies, we use evolutionary game dynamics to investigate PGGs with three types of players [2,3,47]. We first assume that players adjust their strategies by imitating individuals with higher fitness. This strategy updating process can be formulated as the replicator dynamics

$$\begin{aligned}
\tfrac{dx}{dt} &= x(f_C - \bar{f}),\\
\tfrac{dy}{dt} &= y(f_D - \bar{f}),\\
\tfrac{dz}{dt} &= z(f_{CC} - \bar{f}),
\end{aligned} \tag{6}$$

where $\bar{f} = xf_C + yf_D + zf_{CC}$ is the average fitness. From Eq. (6), a strategy with an expected fitness higher than the average will be adopted by more players.

For the standard PGG, Eq. (6) has two sets of equilibria, $(0, y, 1-y)$ with $y \in (0, 1]$ and $(x, 0, 1-x)$ with $x \in [0, 1]$. In the first set of equilibria, the population consists of defectors and conditional cooperators, so no one contributes. In the second set of equilibria, the population consists of cooperators and conditional cooperators, so full contribution can be achieved. Since the fitness of a cooperator (or a defector) is always lower (or higher) than the group average, the trajectories of Eq. (6) will converge to the first set of equilibria (see Fig. 1a). Thus, in the standard PGG, free riding is the only evolutionarily stable state even if there are conditional cooperators in the population.

2.2. PGG with institutional incentives

We now introduce institutional incentives into the PGG. Suppose that there is a third-party institution that can reward and punish players based on their contributions. Specifically, players who contribute c (i.e., full contributors) will be rewarded, and players who contribute 0 (i.e., free riders) will be punished. Thus, if defectors exist in the population, then the institution only rewards cooperators since a conditional cooperator contributes less than c. In contrast, if there is no defector in the population, then conditional cooperators act as cooperators, and the institution rewards both cooperators and conditional cooperators. Analogously, if cooperators exist in the population, then only defectors will be punished. If not, then both defectors and conditional cooperators will be punished.

Regarding the institutional reward, we suppose that the total amount of the reward is R. For the case with defectors (i.e., $y > 0$), the reward is shared among the cooperators. Thus, P_D and P_{CC} are the same as in Eq. (5), and the expected payoff for a cooperator in a randomly formed PGG with n_C cooperators, n_D defectors, and n_{CC} conditional cooperators among the $n-1$ co-players is

$$\begin{aligned}
P_C &= \sum_{n_C=0}^{n-1}\sum_{n_D=0}^{n-1-n_C}\binom{n-1}{n_C}\binom{n-1-n_C}{n_D}x^{n_C}y^{n_D}z^{n_{CC}}\Big(\tfrac{cr}{n}(n_C+1+\tfrac{x}{x+y}n_{CC}) - c + \tfrac{R}{n_C+1}\Big)\\
&= \tfrac{rc(n-1)}{n}\tfrac{x}{x+y} - c + \tfrac{cr}{n} + \tfrac{R}{n}\tfrac{1-(1-x)^n}{x}.
\end{aligned} \tag{7}$$

In this case, Eq. (6) has two sets of equilibria, (i) $(0, y, 1-y)$ with $y \in (0, 1]$ and (ii) $(x^*, 1-x^*, 0)$, where x^* is the solution of $\frac{c(n-r)}{R} = \frac{1-(1-x)^n}{x}$ (see Appendix A for equilibrium calculation). Furthermore, for the case without defectors (i.e., $y = 0$), both cooperators and conditional cooperators are rewarded. In this case, cooperators and conditional cooperators have the same

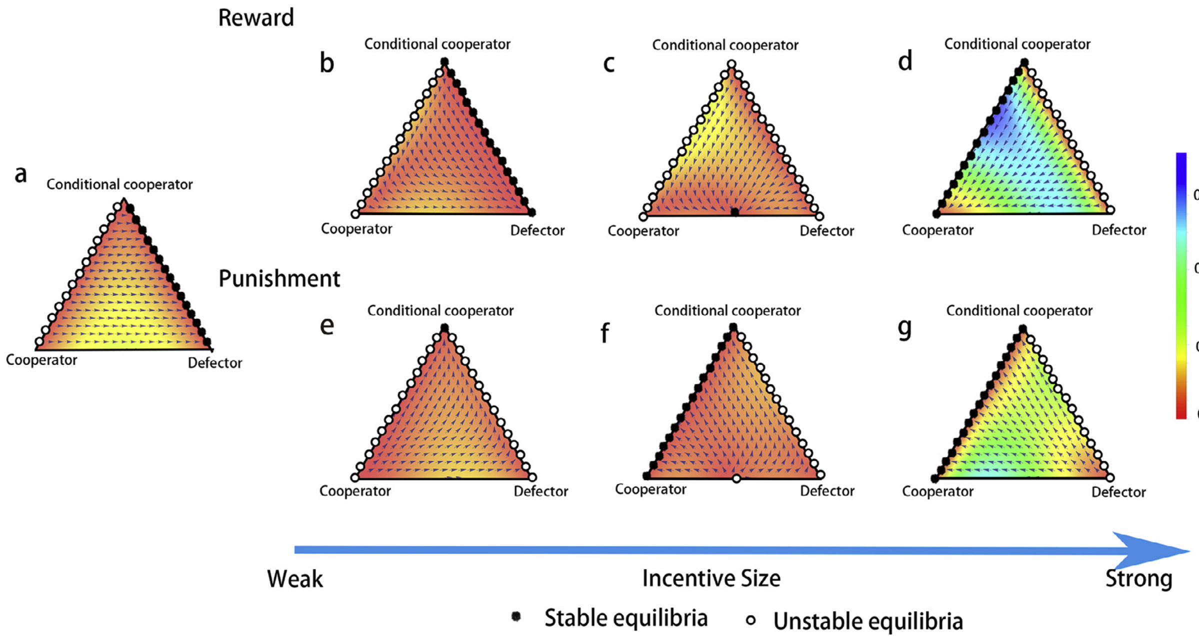

Fig. 1. PGG without decision errors. These figures show phase portraits of the replicator equations Eq. (6) with different types of incentives, where stable equilibria are denoted by solid points and unstable equilibria are denoted by circles. Blue corresponds to fast dynamics, and red corresponds to slow dynamics. Parameters are taken as $n = 4$, $c = 1$, $r = 2$, and $\omega = 0.5$. Furthermore, the incentive sizes are **(a)** $R = P = 0$, **(b)** $R = 0.5$, $P = 0$, **(c)**$R = 1$, $P = 0$, **(d)** $R = 3$, $P = 0$, **(e)** $R = 0$, $P = 0.5$, **(f)** $R = 0$, $P = 1$, and **(g)** $R = 0$, $P = 3$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

expected payoff $P_C = P_{CC} = c(r-1) + \frac{R}{n}$. Thus, (iii) $(x, 0, 1-x)$ with $x \in [0, 1]$ is another set of equilibria for the imitation process because both C and CC players have no incentive to change their strategies. In the first set of equilibria, there is no cooperator in the population, so no one contributes. In the second equilibrium, cooperators and defectors coexist, and the average contribution rate is x^*. Specifically, x^* is increasing in R and r. In the third set of equilibria, there is no defector in the population, so full contribution can be achieved. The stabilities of the three sets of equilibria can be described in terms of the incentive size (see proof in Appendix A). If $R < \frac{c(n-r)}{n}$, then the trajectories converge to the set of free-riding equilibria $(0, y, 1-y)$ with $y \in (0, 1]$ (see Fig. 1b). If $\frac{c(n-r)}{n} < R < c(n-r)$, then the coexistence equilibrium $(x^*, 1-x^*, 0)$ is globally stable. (see Fig. 1c). Finally, if $R > c(n-r)$, then the trajectories converge to the set of cooperative equilibria $(x, 0, 1-x)$ with $x \in [0, 1]$ (see Fig. 1d). It is worth noting that these results are perfectly consistent with the case of PGGs without conditional cooperators [7,16]. This implies that the existence of conditional cooperators does not affect the effectiveness of the reward.

Regarding the institutional punishment, we suppose that the total amount of punishment is P. Analogous to the analysis of IR, we first focus on the case with cooperators (i.e. $x > 0$), where punishment is distributed among the defectors. Thus, P_C and P_{CC} are the same as in Eq. (5), and P_D is rewritten as

$$\begin{aligned} P_D &= \sum_{n_C=0}^{n-1} \sum_{n_D=0}^{n-1-n_C} \binom{n-1}{n_C}\binom{n-1-n_C}{n_D} x^{n_C} y^{n_D} z^{n_{CC}} \frac{cr}{n}\left(n_C + \frac{x}{x+y} n_{CC} - \frac{P}{n_D+1}\right) \\ &= \frac{rc(n-1)}{n}\frac{x}{x+y} - \frac{P}{n}\frac{1-(1-y)^n}{y}. \end{aligned} \tag{8}$$

In this case, Eq. (6) has two sets of equilibria, (i) $(x, 0, 1-x)$ with $x \in (0, 1]$ and (ii) $(1-y^*, y^*, 0)$, where y^* is the solution of $\frac{c(n-r)}{P} = \frac{1-(1-y)^n}{y}$ (see Appendix B for equilibrium calculation). In addition, for the case without cooperators and with defectors (i.e., $x = 0, y > 0$), both defectors and conditional cooperators are punished. In this case, defectors and conditional cooperators have the same expected payoff. Thus, (iii) $(0, y, 1-y)$ with $y \in (0, 1)$ is another set of equilibria for the imitation process. Finally, (iv) (0,0,1) is also an equilibrium, where in this case, the population consists of conditional cooperators only. Analogous to IR, full contribution can be achieved in the first and fourth sets of equilibria, cooperators and defectors coexist at the second equilibrium, and no one contributes at the third set of equilibria. Interestingly, the second and third sets of equilibria must be unstable for any $P > 0$. The trajectories of Eq. (6) will converge to the first set of equilibria for $P > \frac{c(n-r)}{n}$ and move toward the fourth set of equilibria for $0 < P < \frac{c(n-r)}{n}$ (see proof in Appendix B). This implies that the full contribution state is always globally stable. It has been shown that for PGGs without conditional cooperators, free riding is the only stable equilibrium for small incentives $0 < P < \frac{c(n-r)}{n}$, and cooperation and defections are bistable for intermediate incentives $\frac{c(n-r)}{n} < P < c(n-r)$ [7,16]. Thus, the existence of conditional cooperators can help to eliminate defectors and stabilize cooperation even for very small punishment (see Fig. 1e–g).

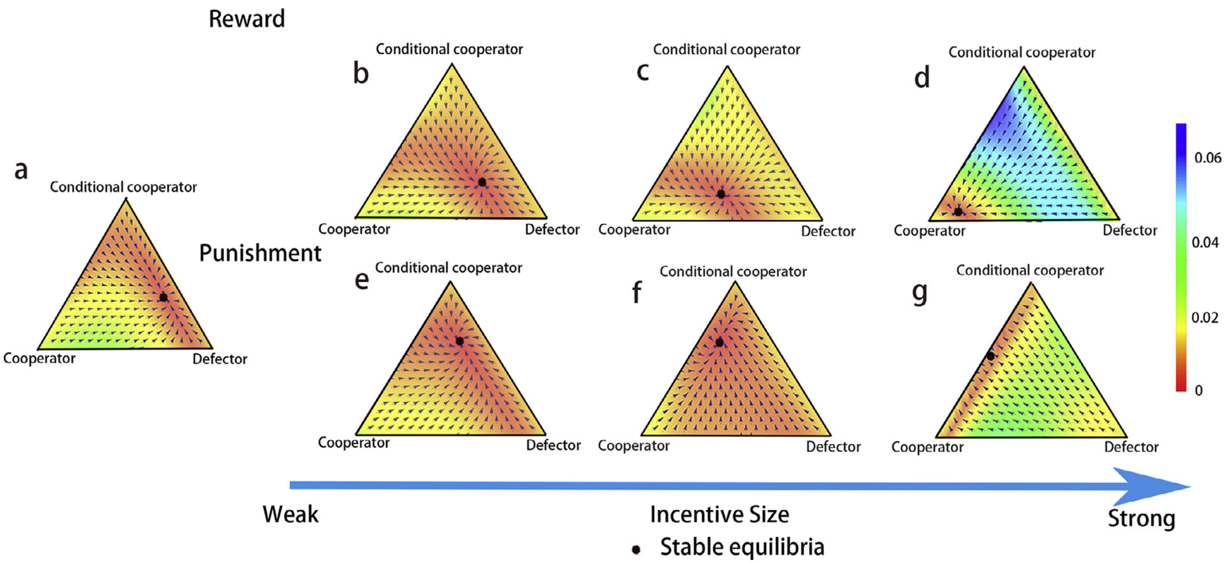

Fig. 2. PGG with decision errors. These figures show phase portraits of the replicator-mutator equations Eq. (9) with different types of incentives, where stable equilibria are denoted by solid points and unstable equilibria are denoted by circles. Blue corresponds to fast dynamics, and red corresponds to slow dynamics. Parameters are taken as $n = 4,\ c = 1,\ r = 2,\ \omega = 0.5$, and $\mu = 0.1$. Furthermore, the incentive sizes in different figures are **(a)** $R = P = 0$, **(b)** $R = 0.5,\ P = 0$, **(c)** $R = 1,\ P = 0$, **(d)** $R = 3,\ P = 0$, **(e)** $R = 0,\ P = 0.5$, **(f)** $R = 0,\ P = 1$, and **(g)** $R = 0,\ P = 3$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2.3. The effect of decision errors

To test the robustness of the above results, we add errors in decision making. In addition to preferential imitation of successful strategies, an individual randomly chooses one of the three strategies C, D, and CC with probability μ. The standard replicator dynamics Eq. (6) are then modified into the following replicator-mutator dynamics:

$$\begin{aligned} \frac{dx}{dt} &= x(f_C - \bar{f}) + \frac{\mu}{3}(1 - 3x), \\ \frac{dy}{dt} &= y(f_D - \bar{f}) + \frac{\mu}{3}(1 - 3y), \\ \frac{dz}{dt} &= z(f_{CC} - \bar{f}) + \frac{\mu}{3}(1 - 3z), \end{aligned} \tag{9}$$

Replicator-mutator dynamics was first introduced in the context of evolution biology, where in this model the selection and mutation terms are separated [47]. For instance, in the first equation of Eq. (9), $x(f_C - \bar{f})$ is the selection term and $\frac{\mu}{3}(1 - 3x)$ is the mutation term.

We find that errors have significant effects on the evolutionary process. First, the boundary states $x = 0$, $y = 0$, or $z = 0$ are no longer equilibria when subjects make mistakes. As a result, full cooperation cannot be sustained. Second, for larger μ, Eq. (9) has a unique and globally stable interior equilibrium, which means that cooperators, defectors, and conditional cooperators will stably coexist (see Fig. 2). Specifically, the interior equilibrium can be expressed explicitly for the standard PGG, where the average contribution rate in this equilibrium is $\frac{1}{2} - \frac{\sqrt{k^2+1}-1}{2k}$ with $k = \frac{\omega c(n-r)}{\mu n}$ (see proof in Appendix C). Note that the contribution rate is always less than half; mutations cannot reverse the disadvantage of cooperation in the standard PGG. Furthermore, the contribution rate decreases to 0 as μ goes to 0.

We next look at the effect of errors in the PGG with incentives. Since it is difficult to obtain an explicit expression for the interior equilibrium, we study the effects of errors on equilibrium through numerical simulations. Fig. 2 shows phase portraits of the replicator-mutator equations Eq. (9) with different types of incentives, and Figs. 3 and 4 compare the contribution rates at the stable equilibria of Eq. (9) for the PGG with reward and punishment for a variety of μ and r. Overall, the effect of the reward is robust to a small μ. As shown in Fig. 3a, the contribution rate for $\mu = 0.01$ is very close to $\mu = 0$ for all R. In contrast, the effect of punishment is sensitive to μ, where the contribution rate drops significantly for smaller P. Nevertheless, punishment still promotes contribution better than reward (see Figs. 3 and 4). For both reward and punishment, the incentive size plays a crucial role in determining the stable contribution rate, where the contribution rate is monotonically increasing in the incentive size. In addition, for smaller R and P, decision errors can help to increase the frequency of cooperators and decrease the frequency of defectors, i.e., the contribution rate is improved. However, for larger R and P, decision errors hinder the positive effect of the incentive institutions, where the stable contribution rate is lower than the case without errors.

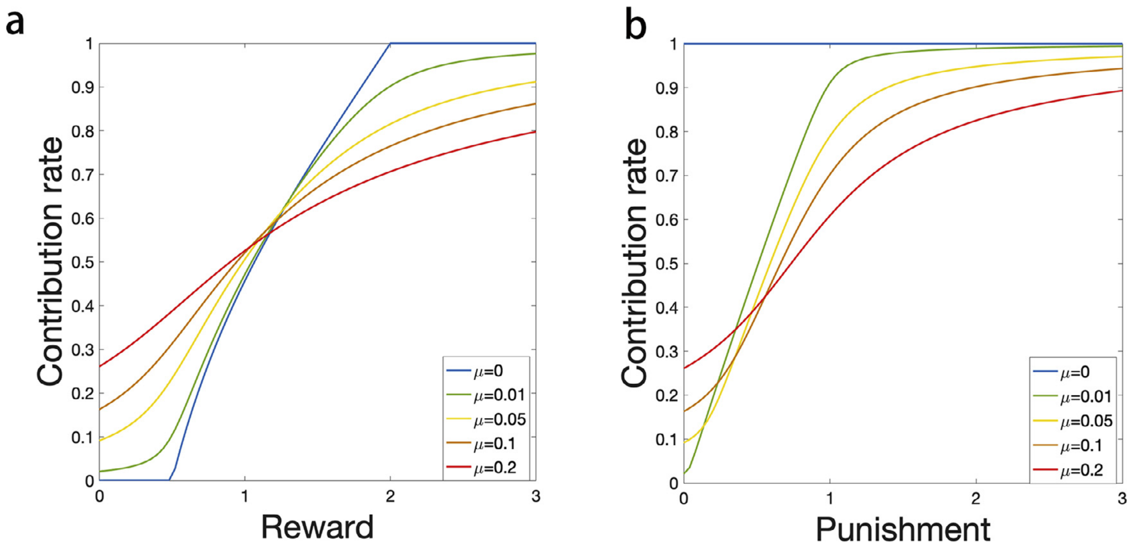

Fig. 3. The effect of decision errors on contribution rate. These figures show the stable contribution rate for PGGs with different μ. Parameters are taken as $n = 4$, $c = 1$, $r = 2$, and $\omega = 0.5$. **(a)** PGG with institutional reward, **(b)** PGG with institutional punishment.

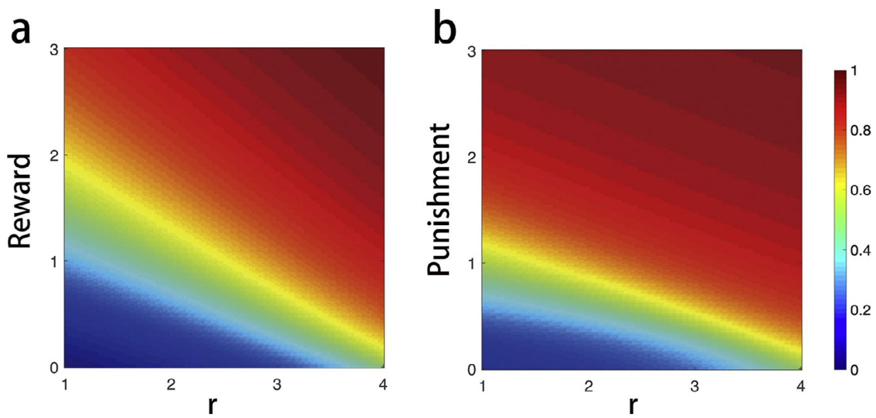

Fig. 4. The effectiveness of reward and punishment for PGG with errors. These figures show the stable contribution rate for PGGs with different r and incentive sizes. Blue corresponds to a low contribution rate, and red corresponds to a high contribution rate. Parameters are taken as $n = 4$, $c = 1$, $\mu = 0.1$, and $\omega = 0.5$. **(a)** PGG with institutional reward, **(b)** PGG with institutional punishment. The contribution rate is increasing in r and incentive size for both reward and punishment. Furthermore, punishment promotes contribution better than reward.. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2.4. The effect of conditional cooperators

To further reveal how conditional cooperators affect the effectiveness of reward and punishment, we compare the stable contribution rate for PGG with and without conditional cooperators. The replicator-mutator dynamics for PGGs with incentives and without conditional cooperators have been studied by Dong et al. [16]. They showed that when subjects make mistakes, reward promotes cooperation better for smaller incentives and punishment promotes cooperation better for intermediate and larger incentives. In the case of reward, our result shows that conditional cooperators have a limited effect on promoting cooperation (see Fig. 5a and b). Games with and without conditional cooperators have the same stable contribution rate for $\mu = 0$, and the stable contribution rates are slightly different for $\mu > 0$. In contrast, the effectiveness of punishment depends crucially on conditional cooperators, where the existence of conditional cooperators can enhance cooperation for smaller incentive sizes. For instance, when $P = 0.5$, subjects do not contribute in the PGG without conditional cooperators, and the average contribution rate is approximately 50% in the PGG with conditional cooperators (see Fig. 5c and d). Overall, PGG with punishment and conditional cooperators leads to the highest cooperation rate than other settings such as PGG with reward or without conditional cooperators.

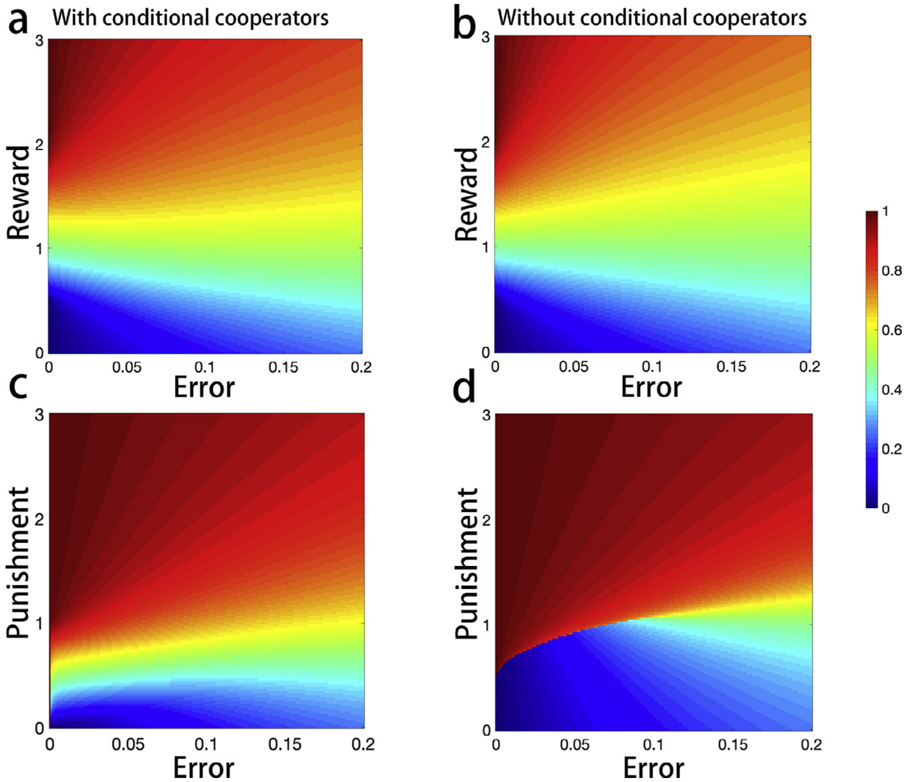

Fig. 5. The role of conditional cooperators. These figures show the stable contribution rate for PGGs with and without conditional cooperators. Blue corresponds to a low contribution rate, and red corresponds to a high contribution rate. Parameters are taken as $n = 4$, $c = 1$, $r = 2$, and $\omega = 0.5$. **(a)** PGG with reward and conditional cooperators, **(b)** PGG with reward and without conditional cooperators, **(c)** PGG with punishment and conditional cooperators, and **(d)** PGG with punishment and without conditional cooperators. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3. Discussion

Recent experiments indicate that most subjects in PGGs are conditional cooperators and tend to choose the group average contribution [25,27,28,32,34–36]. In this paper, we study PGGs with three types of players, namely, cooperators, defectors, and conditional cooperators, and analyze the effects of conditional cooperators on different types of incentives by comparing the average contribution levels at the stable equilibria of evolutionary dynamics. Overall, conditional cooperators cannot lead to a higher contribution level for a standard PGG or a PGG with institutional reward. In contrast, they can enhance the effectiveness of institutional punishment, where a high contribution level can be maintained even for small punishments. As a result, in the PGG with conditional cooperators, punishment always leads to a higher contribution rate than reward, and this result is robust to decision errors.

We now provide an intuitive explanation for why conditional cooperators can promote cooperation in the PGG with punishment and cannot promote cooperation in the standard PGG or the PGG with reward. In the standard PGG, defectors always have a payoff higher than cooperators regardless of the existence of conditional cooperators. Thus, conditional cooperators cannot change the evolutionary trend. In the PGG with reward, if there are cooperators and defectors in the population, then the expected payoff of conditional cooperators must be less than that of defectors because they contribute more but cannot obtain any reward. In this case, the game will degenerate to the situation without conditional cooperators. However, in the PGG with punishment, conditional cooperators perform better than cooperators (because they contribute less) and defectors (because they will not be punished) in general. Therefore, more individuals will adopt a conditional cooperative strategy in the evolutionary process. The reduction in the number of defectors then further increases the average amount of punishment for each defector. Finally, all defectors are eliminated, and full contribution can be maintained. Specifically, for strong punishment, defectors decrease faster than cooperators, so cooperators and conditional cooperators will coexist. In contrast, for weak punishment, most individuals will eventually adopt a conditional cooperative strategy.

In this paper, we simplify the PGG to one shot and assume that conditional cooperators always contribute the average. There are mainly two classes of models for studying the evolution of conditional cooperative strategies in social dilemma games [3]. The first class is based on repeated games, and individuals are assumed to adopt reactive strategies, such as TFT, where their action in a round is determined by what occurred in the previous round [50–54]. The second class simplifies the repeated game to a one-shot game by considering the average behaviour and payoff of an individual during repeated play [55–58]. Dong et al. [37] showed that in a repeated PGG, the contribution of a conditional cooperator will converge to the population average as the expected number of rounds increases. Thus, we use their result to simplify the calculation. In addition, following the intuition explained in the above paragraph, conditional cooperators need not be restricted to contributing the average. In fact, conditional cooperators can promote cooperation in the PGG with punishment as long as their contribution is between those of defectors and cooperators.

We also assume that the incentive institution only rewards full contributors and only punishes free riders. This type of incentive can be characterized as an absolute incentive institution, where the institution punishes (or rewards) all individuals whose contribution is less (or higher) than a predefined threshold [8–13,16,18,21,23,26,38]. We note that our result depends crucially on the threshold setting. If conditional cooperators are rewarded as cooperators (i.e., the reward threshold is low), then the coexistence equilibrium for intermediate reward is no longer stable because conditional cooperators always have a higher payoff than cooperators. Therefore, the existence of conditional cooperators weakens the effectiveness of the reward institution. In contrast, if conditional cooperators are also punished as defectors (i.e., the punishment threshold is high), then payoff for conditional cooperators is always less than that of defectors. The game then degenerates to the situation without conditional cooperators, and cooperation cannot be improved. A more complicated case is that in which the incentive is probabilistic or the amount of the incentive depends on the contribution [7,17,19,25,27,28,48,49]. In this case, rewarding conditional cooperators may help to promote cooperation [28].

In addition, our paper focuses on the cases of reward only and punishment only. It has been shown that a combination of reward and punishment often has a stronger effect on promoting cooperation [9,10,12,16,19,25,27]. Thus, a natural question is how conditional cooperators affect collective cooperation when the institution provides both reward and punishment. This would be a possible direction for future study.

In summary, our study provides a deeper understanding for the role of conditional cooperators on PGGs with institutional incentives. It has been shown that in PGGs with institutional punishment, conditional cooperative strategies can help to dilute the risks of being exploited and punished [25]. Our results reveal that conditional cooperators can not only survive in the evolutionary process but can also help to enhance the effectiveness of punishment. When there are conditional cooperators in the population, institutional punishment is a more reliable way of promoting collaborative efforts than institutional reward: it works even for small incentive size and its effect is robust to errors in decision making.

Authors' contributions

B.Z. and Y.D. designed the study; B.Z., X.A., and Y.D. performed the analysis and wrote the manuscript.

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Declaration of Competing Interest

We declare that we have no competing interests.

Appendix A. Stability analysis for the PGG with reward

For institutional reward, Eq. (6) is written as

$$\begin{aligned}\frac{dx}{dt} &= \omega\Big(-\frac{c(n-r)}{n}\frac{xy}{x+y}+\frac{R}{n}(1-x)(1-(1-x)^n)\Big),\\ \frac{dy}{dt} &= \omega\Big(\frac{c(n-r)}{n}\frac{xy}{x+y}-\frac{R}{n}y(1-(1-x)^n)\Big),\\ \frac{dz}{dt} &= -\omega\frac{R}{n}z(1-(1-x)^n),\end{aligned}\tag{10}$$

where $y > 0$. Note that $\frac{dz}{dt} < 0$ for x, $z \in (0, 1)$, Eq. (10) does not have an interior equilibrium. We now calculate the boundary equilibria of Eq. (10). From $\frac{dz}{dt} = 0$, we have $x = 0$ or $z = 0$. On the on hand, since $\frac{dx}{dt} = \frac{dy}{dt} = 0$ for $x = 0$, the first set of equilibria is $(0, y, 1-y)$ with $y \in (0, 1]$. On the other hand, when $z = 0$, we have $x + y = 1$. Thus, $\frac{dx}{dt} = \frac{dy}{dt} = 0$ if and only if $\frac{c(n-r)}{R} = \frac{1-(1-x)^n}{x}$. Therefore, the second set of equilibrium is $(x^*, 1-x^*, 0)$ such that $\frac{c(n-r)}{R} = \frac{1-(1-x)^n}{x}$. In addition, x^* is unique since that $\frac{1-(1-x)^n}{x} = \sum_{k=0}^{n-1}(1-x)^k$ is a decreasing function of x. Notice that $\frac{c(n-r)}{R}$ is a decreasing function of r and R, x^* is increasing in r and R.

From Eq. (10), we have

$$\frac{dx}{dt} = \Big(R - \frac{c(n-r)}{n}\Big)x + o(x)\tag{11}$$

for x close to 0, and

$$\frac{dy}{dt} = \Big(\frac{c(n-r)-R}{n} + \frac{R}{n}(1-(1-x)^n)\Big)y + o(y) \tag{12}$$

for y close to 0. Furthermore, note that $\frac{dz}{dt} < 0$ for $z \in (0, 1)$ and $x \in (0, 1)$, Eq. (10) does not have an interior fixed point or periodic solutions.

Following Dong et al. [16], the second set of equilibria $(x^*, 1-x^*, 0)$ exists if and only if $\frac{c(n-r)}{n} < R < c(n-r)$. In this case, $\frac{dx}{dt} > 0$ for x is close to 0, $\frac{dy}{dt} > 0$ for y is close to 0, and $\frac{dz}{dt} < 0$. This implies that the first set of equilibria (i.e., $x = 0$) and the third set of equilibria (i.e., $y = 0$) are unstable. Thus, all trajectories will converge to the second set of equilibria.

If $0 < R < \frac{c(n-r)}{n}$, then $\frac{dx}{dt} < 0$ for x close to 0, $\frac{dy}{dt} > 0$ for y close to 0, and $\frac{dz}{dt} < 0$. In this case, the second set of equilibria does not exist, and the third set of equilibria is unstable. Thus, all trajectories will converge to the first set of equilibria.

Finally, if $R > c(n-r)$, then $\frac{dx}{dt} > 0$ for x close to 0, $\frac{dy}{dt} < 0$ for y close to 0, and $\frac{dz}{dt} < 0$. In this case, the second set of equilibria does not exist, and the first set of equilibria is unstable. Thus, all trajectories will converge to the third set of equilibria.

Appendix B. Stability analysis for the PGG with punishment

For institutional punishment, Eq. (6) is written as

$$\begin{aligned}
\frac{dx}{dt} &= \omega\Big(-\frac{c(n-r)}{n}\frac{xy}{x+y} + \frac{P}{n}x(1-(1-y)^n)\Big),\\
\frac{dy}{dt} &= \omega\Big(\frac{c(n-r)}{n}\frac{xy}{x+y} - \frac{P}{n}(1-y)(1-(1-y)^n)\Big),\\
\frac{dz}{dt} &= \omega\frac{P}{n}z(1-(1-y)^n),
\end{aligned} \tag{13}$$

where $x > 0$. Note that $\frac{dz}{dt} > 0$ for $y, z \in (0, 1)$, Eq. (13) does not have an interior equilibrium. We now calculate the boundary equilibria of Eq. (13). From $\frac{dz}{dt} = 0$, we have $y = 0$ or $z = 0$. On the on hand, since $\frac{dx}{dt} = \frac{dy}{dt} = 0$ for $y = 0$, the first set of equilibria is $(x, 0, 1-x)$ with $x \in (0, 1]$. On the other hand, when $z = 0$, we have $x + y = 1$. Thus, $\frac{dx}{dt} = \frac{dy}{dt} = 0$ if and only if $\frac{c(n-r)}{P} = \frac{1-(1-y)^n}{y}$. Therefore, the second set of equilibrium is $(1-y^*, y^*, 0)$ such that $\frac{c(n-r)}{P} = \frac{1-(1-y)^n}{y}$. In addition, y^* is unique since that $\frac{1-(1-y)^n}{y} = \sum_{k=0}^{n-1}(1-y)^k$ is a decreasing function of y.

From Eq. (13), we have

$$\frac{dx}{dt} = \Big(-\frac{c(n-r)}{n} + \frac{P}{n}(1-(1-y)^n)\Big)x + o(x) \tag{14}$$

for x close to 0, and

$$\frac{dy}{dt} = \Big(\frac{c(n-r)}{n} - P\Big)y + o(y) \tag{15}$$

for y close to 0.

Note that $\frac{dz}{dt} > 0$ for $z \in (0, 1)$ and $y \in (0, 1)$, Eq. (13) does not have an interior fixed point or periodic solutions. Furthermore, the second set of equilibria $(1-y^*, y^*, 0)$ must be unstable [16], and a trajectory with initial value in (or close to) the third set of equilibria (i.e., $x = 0$ and $y > 0$) will move toward (0,0,1) since $\frac{dz}{dt} > 0$.

If $P > \frac{c(n-r)}{n}$, then $\frac{dy}{dt} < 0$ for y close to 0. Thus, all trajectories will converge to the first set of equilibria or (0,0,1).

In contrast, if $0 < P < \frac{c(n-r)}{n}$, then $\frac{dy}{dt} > 0$ for y close to 0, which means that the first set of equilibria is also unstable. In this case, (0,0,1) is the only stable equilibrium because $\frac{dz}{dt} > 0$ always holds. This implies that all trajectories will converge to the fourth set of equilibria (0,0,1).

Appendix C. Stability analysis for the standard PGG with errors

For the standard PGG, Eq. (9) can be written as

$$\begin{aligned}
\frac{dx}{dt} &= -\frac{c(n-r)}{n}\frac{xy}{x+y} + \frac{\mu}{3}(1-3x),\\
\frac{dy}{dt} &= \frac{c(n-r)}{n}\frac{xy}{x+y} + \frac{\mu}{3}(1-3y),\\
\frac{dz}{dt} &= \frac{\mu}{3}(1-3z).
\end{aligned} \tag{16}$$

Eq. (16) has a unique interior equilibrium

$$(x, y, z) = \Big(\frac{1}{3} - \frac{\sqrt{k^2+1}-1}{3k}, \frac{1}{3} + \frac{\sqrt{k^2+1}-1}{3k}, \frac{1}{3}\Big), \tag{17}$$

where $k = \frac{\omega c(n-r)}{\mu n}$. To analyse the stability of this equilibrium, we calculate the eigenvalues of the Jacobian matrix in this equilibrium. The real parts of the eigenvalues are $-\mu$ and $-\mu(2 - k(\frac{1}{3} - \frac{\sqrt{k^2+1}-1}{3k})^2)$, where $k(\frac{1}{3} - \frac{\sqrt{k^2+1}-1}{3k})^2$ takes its maximum $\frac{6-4\sqrt{2}}{9}$ at $k = 1$. Since all the eigenvalues have negative real parts, the interior equilibrium is locally stable. Note that $\frac{dz}{dt}$ is independent of x and y, and Eq. (16) does not have periodic solutions. Thus, the interior equilibrium is also globally stable. In addition, the average cooperation rate in this equilibrium is $\frac{1}{2} - \frac{\sqrt{k^2+1}-1}{2k}$.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.amc.2020.125600

References

[1] E. Pennisi, How did cooperative behavior evolve? Science 309 (2011) 93.
[2] M.A. Nowak, Evolutionary Dynamics: Exploring the Equations of Life, Harvard University Press, 2006.
[3] K. Sigmund, The Calculus of Selfishness, Princeton University Press, 2010.
[4] D. Balliet, L.B. Mulder, P.A.M.V. Lange, Reward, punishment, and cooperation: a meta-analysis, Psychol. Bull. 137 (2011) 594–615.
[5] F. Guala, Reciprocity: weak or strong? What punishment experiments do (and do not) demonstrate, Behav. Brain. Sci. 35 (2012) 1–15.
[6] M. Perc, J.J. Jordan, D.G. Rand, Z. Wang, S. Boccaletti, A. Szolnoki, Statistical physics of human cooperation, Phys. Rep. 687 (2017) 1–51.
[7] R. Cressman, J. Song, B. Zhang, Y. Tao, Cooperation and evolutionary dynamics in the public goods game with institutional incentives, J. Theor. Biol. 299 (2012) 144–151.
[8] T. Sasaki, A. Brännström, U. Dieckmann, K. Sigmund, The take-it-or-leave-it option allows small penalties to overcome social dilemmas, Proc. Natl. Acad. Sci. USA 109 (2012) 1165–1169.
[9] X. Chen, T. Sasaki, A. Brännström, U. Dieckmann, First carrot, then stick: how the adaptive hybridization of incentives promotes cooperation, J. R. Soc. Interface 12 (2015) 20140935.
[10] T. Sasaki, S. Uchida, X. Chen, Voluntary rewards mediate the evolution of pool punishment for maintaining public goods in large populations, Sci. Rep. 5 (2015) 08917.
[11] L. Gao, Z. Wang, R. Pansini, Y. Li, R. Wang, Collective punishment is more effective than collective reward for promoting cooperation, Sci. Rep. 5 (2015) 17752.
[12] I. Okada, H. Yamamoto, F. Toriumi, T. Sasaki, The effect of incentives and meta-incentives on the evolution of cooperation, PLoS Comput. Biol. 11 (2015) e1004232.
[13] A. Szolnoki, M. Perc, Antisocial pool rewarding does not deter public cooperation, Proc. R. Soc. Lond. B 282 (2015) 20151975.
[14] A. Szolnoki, M. Perc, Second-order free-riding on antisocial punishment restores the effectiveness of prosocial punishment, Phys. Rev. X 7 (2017) 041027.
[15] H.X. Yang, X. Chen, Promoting cooperation by punishing minority, Appl. Math. Comput. 316 (2018) 460–466.
[16] Y. Dong, T. Sasaki, B. Zhang, The competitive advantage of institutional reward, Proc. R. Soc. Lond. B 286 (2019) 20190001.
[17] N. He, X. Chen, A. Szolnoki, Central governance based on monitoring and reporting solves the collective-risk social dilemma, Appl. Math. Comput. 347 (2019) 334–341.
[18] Y. Fang, T. Benko, M. Perc, H. Xu, Q. Tan, Synergistic third-party rewarding and punishment in the public goods game, Proc. R. Soc. A 475 (2019) 20190349.
[19] S. Huang, Y. Dong, B. Zhang, The optimal incentive in promoting cooperation: punish the worst and do not only reward the best, Singap. Econ. Rev. (2020), doi:10.1142/S0217590820500071. Online
[20] T. Yamagishi, The provision of a sanctioning system as a public good, J. Pers. Soc. Psychol. 51 (1986) 110–116.
[21] M. Sutter, S. Haigner, M.G. Kocher, Choosing the carrot or the stick? Endogenous institutional choice in social dilemma situations, Rev. Econ. Stud. 77 (2010) 1540–1566.
[22] L. Putterman, J.L. Tyran, K. Kamei, Public goods and voting on formal sanction schemes, J. Public. Econ. 95 (2011) 1213–1222.
[23] A. Traulsen, T. Röhl, M. Milinski, B. Bio., An economic experiment reveals that humans prefer pool punishment to maintain the commons, Proc. R. Soc. Lond. 279 (2012) 3716–3721.
[24] R. Cressman, J. Wu, C. Li, Y. Tao, Game experiments on cooperation through reward and punishment, Biol. Theory 8 (2013) 158–166.
[25] J. Wu, C. Li, B. Zhang, R. Cressman, Y. Tao, The role of institutional incentives and the exemplar in promoting cooperation, Sci. Rep. 4 (2014) 6421.
[26] B. Zhang, C. Li, H. De Silva, P. Bednarik, K. Sigmund, The evolution of sanctioning institutions: an experimental approach to the social contract, Exp. Econ. 17 (2014) 285–303.
[27] Y. Dong, B. Zhang, Y. Tao, The dynamics of human behavior in the public goods game with institutional incentives, Sci. Rep. 6 (2016) 28809.
[28] C. Yang, B. Zhang, G. Charness, C. Li, J.W. Lien, Endogenous rewards promote cooperation, Proc. Natl. Acad. Sci. USA 115 (2018) 9968–9973.
[29] A. Szolnoki, M. Perc, Conformity enhances network reciprocity in evolutionary social dilemmas, J. R. Soc. Interface 12 (2015) 20141299.
[30] F. Shu, Y. Liu, X. Liu, X. Zhou, Memory-based conformity enhances cooperation in social dilemmas, Appl. Math. Comput. 346 (2019) 480–490.
[31] C. Keser, F. van Winden, Conditional cooperation and voluntary contributions to public goods, Scand. J. Econ. 102 (2000) 23–39.
[32] U. Fischbacher, S. Güchter, E. Fehr, Are people conditionally cooperative? Evidence from a public goods experiment, Econ. Lett. 71 (2001) 397–404.
[33] R. Kurzban, D. Houser, Experiments investigating cooperative type in humans: a complement to evolutionary theory and simulations, Proc. Natl. Acad. Sci. USA 102 (2005) 1803–1807.
[34] U. Fischbacher, S. Güchter, Social preferences, beliefs, and the dynamics of free riding in public goods experiments, Am. Econ. Rev. 100 (2010) 541–556.
[35] A. Chaudhuri, Sustaining cooperation in laboratory public goods experiments: a selective survey of the literature, Exp. Econ. 14 (2011) 47–83.
[36] K.E. Hauge, K.A. Brekke, K. Nyborg, J.T. Lind, Sustaining cooperation through self-sorting: the good, the bad, and the conditional, Proc. Natl. Acad. Sci. USA 116 (2019) 5299–5304.
[37] Y. Dong, C. Li, Y. Tao, B. Zhang, Evolution of conformity in social dilemmas, PLoS One 10 (2015) E0137435.
[38] Y. Kamijo, T. Nihonsugi, A. Takeuchi, Y. Funaki, Sustaining cooperation in social dilemmas: comparison of centralized punishment institutions, Games. Econ. Behav. 84 (2014) 180–195.
[39] A. Traulsen, C. Hauert, H. De Silva, M.A. Nowak, K. Sigmund, Exploration dynamics in evolutionary games, Proc. Natl. Acad. Sci. USA 106 (2009) 709–712.
[40] K. Sigmund, H. DeSilva, A. Traulsen, C. Hauert, Social learning promotes institutions for governing the commons, Nature 466 (2010) 861–863.
[41] A. Traulsen, D. Semmann, R.D. Sommerfeld, H.J. Krambeck, M. Milinski, Human strategy updating in evolutionary games, Proc. Natl. Acad. Sci. USA 107 (2010) 2962–2966.
[42] H. Ito, J. Tanimoto, Scaling the phase- planes of social dilemma strengths shows game-class changes in the five rules governing the evolution of cooperation, R. Soc. Open. Sci. 5 (2018) 1–9.
[43] J. Tanimoto, Fundamentals of Evolutionary Game Theory and its Applications, Springer, 2015.

[44] J. Tanimoto, Evolutionary Games with Sociophysics: Analysis of Traffic Flow and Epidemics, Springer, 2019.
[45] J. Tanimoto, H. Sagara, Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game, BioSystems 90 (2007) 105–114.
[46] Z. Wang, S. Kokubo, M. Jusup, J. Tanimoto, Universal scaling for the dilemma strength in evolutionary games, Phys. Life Rev. 14 (2015) 1–30.
[47] J. Hofbauer, K. Sigmund, Evolutionary Games and Population Dynamics, Cambridge University Press, 1998.
[48] J. Andreoni, L.L. Gee, Gun for hire: delegated enforcement and peer punishment in public goods provision, J. Public Econ. 96 (2012) 1036–1046.
[49] X. Qin, S. Wang, Using an exogenous mechanism to examine efficient probabilistic punishment, J. Econ. Psychol. 39 (2013) 1–10.
[50] M.A. Nowak, K. Sigmund, Tit for tat in heterogeneous populations, Nature 355 (1992) 250–253.
[51] M.A. Nowak, K. Sigmund, A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's dilemma game, Nature 364 (1993) 56–58.
[52] W.H. Press, F.D. Dyson, Iterated Prisoner's dilemma contains strategies that dominate any evolutionary opponent, Proc. Natl. Acad. Sci. USA 109 (2012) 10409–10413.
[53] C. Hilbe, M.A. Nowak, K. Sigmund, The evolution of extortion in iterated Prisoner's dilemma games, Proc. Natl. Acad. Sci. USA 110 (2013) 6913–6918.
[54] C. Hilbe, B. Wu, A. Traulsen, M.A. Nowak, Cooperation and control in multiplayer social dilemmas, Proc. Natl. Acad. Sci. USA 111 (2014) 16425–16430.
[55] M.A. Nowak, A. Sasaki, C. Taylor, D. Fudenberg, Emergence of cooperation and evolutionary stability in finite populations, Nature 428 (2004) 646–650.
[56] L.A. Imhof, D. Fudenberg, M.A. Nowak, Evolutionary cycles of cooperation and defection, Proc. Natl. Acad. Sci. USA 102 (2005) 10797–10800.
[57] H. Brandt, K. Sigmund, The good, the bad and the discriminator—Errors in direct and indirect reciprocity, J. Theor. Biol. 239 (2006) 183–194.
[58] A. Szolnoki, M. Mobilia, L. Jiang, B. Szczesny, A.M. Rucklidge, M. Perc, Cyclic dominance in evolutionary games: a review, J. R. Soc. Interface 11 (2014) 20140735.